On Fermat's Last Tantrum

Version: 9 September 2018

I was struggling with combinatorics. You know, that science where there appears to be little method and less specificity in its naming conventions. "Is limiting each thing to one name too much to ask?," I was asking myself when once again, my doorway filled with tobacco smoke. Stinky tobacco smoke. Now, I smoke a pipe myself. But I choose mixes with a nice room note. This stench, I knew, heralded the return of Herr Doktor Niemand, than whom no one is more annoying. I braced myself. But not quickly enough.

Or rather, I was startled into distraction when a little book-keeper of a man rushed angrily into the room, waving a large phone book, exclaiming: "C'est merdique." Herr Doktor Niemand was right behind him. As I raised a hand in greeting, the book-keeper smacked me with the phone book.

"Ow," I said.

"Mer. Di. Que," he repeated, loudly and at length, as he hit me again.

"What?" I said. "Stop that. What's merdique?"

The man drew his arm back to swing at me once more. But Niemand took the book away from him, saying: "It is French for scheisslich."

"I know that," I said. "What is scheisslich?"

"Msr. De Fermat, considers this inadequate." he replied, tossing me the phone book. Which was not a phone book. It was a proof, by a Professor Wiles, of Fermat's Last Theorem. A rather lengthy proof.

"Why is he hitting me with it?" I asked.

"He's upset," said Niemand. "Anyone can see that."

"Does he speak English?" I asked.

"Un peut," replied Fermat.

"Why are you upset?" I asked.

"Parceque this proof is incomprehensible," he replied.

I thumbed through it a bit.

"I agree," I said. "But I'll put that down to my gross ignorance of higher mathematics."

"I, too, am ignorant of all this, this abstraction," he said. "But Herr Niemand assures me that it is valid."

"So what's the problem," I asked.

"The problem is the very insult of this proof. It is a scandal of mathematics. When I said my proof of this would not fit in the margin of my Diophantus, I did not say that it would not fit in all of the margins in my library. I simply was too comfortable to take a piece of paper from my escritoire."

"One piece?" I asked.

"One side would suffice." he said, smiling.

"So publish it," I said.

"I am dead," he replied.

"Yes. Well, there's that," I admitted. "So why are you here?"

"I told him you would publish it," explained the doktor, who was still putrifying the room with his pipe.

"Oh, sure," I said. "I'll just send it in under his name and I'm sure that Stanford or whoever will leap at the chance."

"Oh, non," said Fermat. "It will be under your name. Parceque it will be your proof."

"I told him how talented you were," said Niemand.

"Oh, yeah," I said. "It took me eight days to get out of a trisected angle trap while I struggled to figure out how Euclid would know why it wasn't trisected. And he would have known. And I finally figured out how he knew and went back to my normal life. Which I would prefer not to leave again."

"This is no more difficult," smiled Niemand.

"Forget it," I said.

"We will give you a hint," said Fermat. "You will be famous."

"I'd rather be rich," I said. "Instead, I'll be ridiculed."

"So you accept," said Niemand. Apparently, he knows me.

"So what's my hint?" I asked.

Fermat gestured at Niemand, who said: "It is not a triangle."

And they were gone.

By way of disclosure, I was already interested in Fermat's Last Theorem. The theorem is that, for n > 2, there are no integer solutions for:

$$x^n + y^n = z^n$$

Pretty simple. And, as I had expected, Fermat had had a brief proof of similar simplicity. Which he hadn't shown me. But I had my hint. So I started with no triangle.

I started with n=2.

I asked myself what second square is added to a first square in order to make a third. Let's say you have one square a^2. Then for some n, you have to add $2an + n^2$. Then $a^2 + 2an + n^2 = b^2$, your next square up. If you have 3^2, you add $2 \cdot 3 \cdot 2 + 2^2$ or 4^2 and you get 5^2. I figure this out with little square boxes on a sheet of paper using 3,4,5. The question, of course, is when is $(2an + n^2)$ the square of some integer b.

There are easier ways to do this. But I was full of Euclid at the time. And $(2an + n^2)$ struck me as the rectangle n•(n+2a) which made me think of Euclid III.36 or "If from point outside ⊙, one line is drawn to touch ⊙ and one to cut it, the square of the first equals the rectangle of the second and its outside segment."

So I drew a circle and built a right triangle. One side was the diameter (2r). One side was the tangent that made the square. And the hypotenuse I rearranged so that the chord was n and the outside bit was 2a. And this gave me:

$$(2r)^2 + (\sqrt{(4a^2+2an+ n^2 - 4r^2)})^2 = (2a + n)^2$$

And the question becomes, for what values of r is that middle term an integer. You may be wondering, "Where is this idiot going with this?" I eventually asked the same thing, only in a nicer way. But before I dropped this line of inquiry, which had me looking for analogues of III.36 in n dimensions, I discovered some pretty things about the relation of the r's to the magnitude of side a which gives all right triangles on a. We'll skip that as no text of mathematics is complete without an exercise for the reader.

I moved on from my n-dimensional delerium when Niemand dropped in to see how I was doing. I was rather proud, baby that I am, of my Euclidean diagram. I showed it to him.

"That is a triangle," he said. "And I told you: it is not a triangle."

I pointed at the circle.

"That is also no triangle, " he said. "But it will help you not at all."

I must have looked hurt. Niemand patted me on the shoulder.

"It is not a triangle," he said with a smile and was gone.

Okay. Fine. I gave the problem a good, long rest. Until one day ...

The hell it's not a triangle, I told myself one day. We have

$$x^n + y^n = z^n$$

So we have, if n=1

$$x + y = z$$

And if you give me three magnitudes, I thought, I have a triangle. So let's change x,y,z into sides a,b,c. Then, ignoring that last uninspiring equation, we have

$$a + b > c \text{ (Euclid I.20)}$$

And, triangle-wise, as any trigonometer (or -trix) knows:

$$c^2 = a^2 + b^2 - 2ab\bullet\cos C$$

OR if $a+b > c$, Fermat's equation becomes:

$$a^n + b^n = (\sqrt{(a^2 + b^2 - 2ab\bullet\cos C)})^n$$

Well, cosC, being what it is, puts c right out of the integer solution ballpark. Which means, if we're going to prove or disprove Fermat, we have cosC = 0 or 1 and:

$$a^n + b^n = (\sqrt{(a^2 + b^2)})^n \text{ or } (\sqrt{(a^2 + b^2 - 2ab)})^n$$

First case only works for n=2 as this is only the space of right triangles. By experimentation, you quickly see that a^n and b^n are quickly outpaced by c^n and can no longer serve as sides. This is easily shown with the Binomial Theorem with fractional exponents. ... (

Let me pause to elucidate why the hypotenuse to the $n > 2$ must always run away from the sides. Take any semicircle and let the diameter be the hypotenuse c of our triangle. Now think of all the right triangles of sides a, b and c in this semicircle. The value of b is on the interval of [a,0). So c is on the interval [$\sqrt{2}$a,a). Now for all of these $a^2 + b^2 = c^2$, from the $a^2 + a^2 = (\sqrt{2}\times a)^2$ to the (almost $b)^2$ + (almost $0)^2 = c^2$. You can see that the $\sqrt{2}$ in the first case goes almost, but not quite, to 1, in the second. This factor when raised to n > 2 ensures the hypotenuse runs away from the sum of the sides in $a^n + b^n = c^n$ or, as Fermat would have it, $x^n + y^n = z^n$.

Consider the semicircle of diameter 5. It happens to have a right triangle in it with sides 3,4,5. If we cube these sides, the hypotenuse exceeds the sides by 34. If we slide a up so that c = $\sqrt{2}\times$a, the hypotenuse³ exceeds the sides³ by 36 and change. Then slide b down to almost 0, a almost becomes c, but there is always this factor from $\sqrt{2}$ down to just greater than 1, limitly Calculusly speaking, which when raised to n = 3,4,5,,, forces z^n to always exceed $x^n + y^n$.

I add this elucidation, as my amends for the mischevious use of the Binomial Theorem with fractional etc. in the ChangeLog below, are only amends if you think about a sliding a in a, a+x, and a+x+1. Which no one would unless they were told. And I didn't tell. So when I realized my amends were not really amends, I added this elucidation, which I hope shall suffice and which has taught me not to play with the Binomial Theorem in jest, even though it would work just fine if you had sufficient time on your hands. We should be serious and spell things out so that all may share in the joy of understanding.

Of course, now, if we consider the three sides of a right triangle as (a), a fraction of a (Fa), and $(1 + F^2)^{1/2}$a, using the Binomial Theorem, we see plainly that for n>3, $1^n + F^n < (1 + F^2)^{n/2}$. All ideas have their right place.

Let us return to our argument.

) ... In the second case, $c = |a - b|$. These arguments extend, with equal potency, to obtuse angles. And our triangle ... wait for it ... is not a triangle.

Well, in our $a^n + b^n = c^n$, a and b stand in some relation to c. It clearly is not $a+b > c$. Let $a+b = c$ and we have

$$(a+b)^n = c^n$$

And consequently:

$$a^n + b^n < (a+b)^n = c^n$$

So we can say that Fermat is batting two for two. And all but the slowest of you have long gone on to:

$$\text{If } a+b < c, \text{ then } a^n + b^n < (a+b)^n < c^n$$

Fermat's batting average maxes out for this game and we write:

QED

Except no one has checked my work here. No fat lady has sung. Niemand came back to tell me to forget the whole thing as Fermat had died again and wouldn't explain beyond that. I tried to get Niemand to look at it before he exited in a cloud of smoke. But he wasn't interested and left me with my unaffirmed work in my hand.

So I can only appeal to those who have read this far:

"What, if anything, have I missed here? QED or non?"

Comments, criticism, and postal money-orders welcome at:
r.earle.harris@gmx.com

Acknowledgements

I acknowledge my debt to the qualities of mind expressed by Euclid of Alexandria, Isaac Todhunter, and Edward Kimball.

Apologia

The above dramatic conceit is intended only as entertainment. Fermat would never hit anyone with a phone book. Nor would he, nor I, speak ill of Professor Wiles and his work (which I have never seen and would be too slow and thick to understand). If there is any subtext here (and with me, there usually isn't) it is that of Thoreau's: "Simplify, simplify, simplify."

Changelog

30mar18:
initial release
31mar18:
And this only works for n=1,2. By experimentation, you quickly
=>
And this only works for n=2 as this is only the space of right triangles. By experimentation, you quickly see that a^n and b^n are
01apr18:
added initial release date to changelog
acknowledgements and apologia
----
quickly outpaced by c^n and can no longer serve as sides. And
=>

quickly outpaced by c^n and can no longer serve as sides, even if cosC=1. And

----

$$a^n + b^n = (\sqrt{(a^2 + b^2)})^n$$

And this only works for n=2 as this is only the space of right triangles. By experimentation, you quickly see that a^n and b^n are =>

$$a^n + b^n = (\sqrt{(a^2 + b^2)})^n \text{ or } (\sqrt{(a^2 + b^2 + 2ab)})^n$$

First case only works for n=2 as this is only the space of right triangles. By experimentation, you quickly see that a^n and b^n are

----

02apr18

$a^n + b^n = (\sqrt{(a^2 + b^2)})^n$ or $(\sqrt{(a^2 + b^2 + 2ab)})^n$

=>

$a^n + b^n = (\sqrt{(a^2 + b^2)})^n$ or $(\sqrt{(a^2 + b^2 - 2ab)})^n$

----

even if cosC=1. And our triangle

=>

In the second case, $c = |a - b|$. And our triangle

03apr18

cosC = 0 => cosC = 0 or 1

05apr18

Binomial Theorem remark

08apr18

That last edit was a bit Puckish, in the style of Lewis Carroll, Niemand's creator. I add this as amends for my behavior:

The quickest way to see the runaway of the hypotenuse ^{n}over the $\sum$ side^n is to let the sides be a, a+x, and the hypotenuse a+(x+1). Expand the last two with the Binomial Theorem. Subtract the sides

from the hypotenuse and you can see how quickly the latter overwhelms the former. There. I feel better.
21apr18
Remark on obtuse angles.
03jun18
Added pause of elucidation
06jun18
Binomial Thm in elucidation
09sep18
stupid font fix on page 4