Euclid – Todhunter – Synopsis

Book I

Book I
Definitions
1.1 A **point** is position without magnitude.
1.2 A **line** is length without breadth.
1.3 The extremities and intersections of lines are points.
1.4 A **straight line** lies evenly between its extremities.
1.5 A **surface** is length and breadth.
1.6 The boundaries of surfaces are lines.
1.7 A **plane** is a surface where a line joining two of its points lies entirely on the surface.
1.8 A **plane angle** (∡) is the inclination of two lines to one another meeting on a plane.
1.9 A **plane rectilinear ∡** is the plane ∡ of two straight lines. Their intersection is the angle's vertex.
(Note: straight lines will be denoted “lines” and curved lines as “curves” from this point.)
1.10 When a line meets another so as to make two equal ∡, it is **perpendicular** (⊥) to the other creating two **right angles** (⊾)
1.11 An **obtuse ∡** is greater than a ⊾.
1.12 An **acute ∡** is less than a ⊾.
1.13 A **plane figure** is any shape enclosed by lines or curves and these are its **boundary**.
1.14 If the boundary is composed of lines, it is a **rectilinear figure** (n-gon of n sides) and the lines are its sides.
1.15 A **circle** (○) is a plane figure bounded by all points equal from its center.
1.16 A line from a ○'s center to its boundary is its **radius**.
1.17 A radius extended to the opposite boundary is the ○'s **diameter**.
1.18 A **semicircle** is bounded by diameter and the remaining boundary.
1.19 A **circular segment** is bounded by a line and the circular boundary it cuts off.
1.20 A **triangle** (△) is bounded by three straight lines. Any angular point can be its **vertex** and the opposite side is the **base**.
1.21 A **quadrilateral** (4-gon) is bounded by four lines. A line between opposite vertices is the diagonal.
1.22 A **polygon** is a figure bounded by more than 4 lines.
1.23 An **equilateral △** has three equal sides.
1.24 An **isosceles △** has two equal sides.
1.25 A **scalene △** has three unequal sides.
1.26 A **right △** (⊾△) has one ⊾. Its opposite side is the **hypotenuse**.
1.27 An **obtuse △** has one obtuse angle.
1.28 An **acute △** has three acute angles.
1.29 **Parallel** (∥) lines are two coplanar lines which, extended, never intersect.
1.30 A **∥gm** is a 4-gon of opposing parallel sides.
1.31 A **square** is an equilateral 4-gon with a ⊾.
1.32 A **rectangle** is a ∥gm with a ⊾.
1.33 A **rhombus** is an equilateral 4-gon with no ⊾.
1.34 A **rhomboid** is a 4-gon with equal opposing sides and no ⊾.
1.35 A **trapezium** is a 4-gon with two ∥ sides.

Postulates
1.A line may be drawn through any two points.
2. A line may be indefinitely extended.
3. Any point and any line from it may be used to create a circle

Axioms
1. Things equal to same thing are equal to each other.
2. Equals added to equals make equals.
3. Equals taken from equals make equals.
4. Equals added to unequals make unequals.
5. Equals taken from unequals make unequals.
6.Things double the same thing are equals.
7, Things half the same thing are equals.
8. The whole is greater than its parts.
9. Magnitudes which can be made to coincide are equal.
10. Two lines cannot include a space. They share 0, 1, or all points in common.
11. All ⊾ are equals.
12. If a line meet two lines, such that upon the same side it creates two equal ∡, together less than two ⊾, the two lines, extended on that side must meet.

Triangles – Equal
1.4 2 equal sides w/equal int ∡
1.8 3 equal sides
1.26 2 equal ∡s w/one equal side

Triangles – Isosceles
1.5 if isos then int. and ext. base ∡s equal
1.6 △ w/2 equal ∡s, ∡s opp. sides equal

Constructions
1.1 equilat △ on AB
1.2 copy AB to C
1.3 copy AB < CD to C
1.9 bisect ∡
1.10 bisect AB
1.11 produce AB ⊥ CD
1.12 from A create ⊥ to BC
1.22 construct △ from 3 lines
1.23 @C on AB copy ∡D
1.31 @A create BC ∥ DE
1.42 Given △ and other ∡ create ∥gm = △
1.44 Given △, AB, ∡ create ∥gm on AB w/∡ = △
1.45 Given ∡ and rectilinear figure, create ∥gm w/∡ = figure
1.46 Given AB create AB^2

Triangles
1.5.C1 equilateral is equiangular
1.6.C1 converse of 1.5.C1
1.7 2 △s same base,if 2 sides one end of base equal, other 2 sides equal
1.16 ext ∡ of side > either opp int ∡
1.17 any 2 < 2⊾
1.18 greater sides have greater opp ∡s
1.19 converse of 1.18
1.20 any 2 sides greater than 3d
1.21 △ built inside △ on same base has smaller sides, greater ∡
1.24 2 △ w/equal adj sides, greater int ∡ has greater base
1.25 converse of 1.24
1.32 ext ∡ = sum of int opp ∡s and all int s = 2⊾
1.47 On ⊾△, square on hypotenuse equals squares on other two sides
1.48 Converse of 1.47

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Books I - III

Lines

1.13 ∡s one side of line cut by line = 2⊾

1.14 2 lines meet a third @A and making adj ∡s = 2⊾ are same line

1.15 Intersecting lines, opp. ∡s equal

1.15.C1 The 4 opp ∡s = 4⊾

1.15.C2 All lines at same point create 4⊾.

1.27 If line cuts 2 lines and alt. ∡s equal, the 2 lines are ‖

1.28 2 lines ‖ if cutting line makes int. ∡s on same side equal 2⊾ -or- ext. ∡s = opp ∡s on same side

1.29 Line cutting 2 ‖ lines creates ∡ relations of 1.27, 1.28

1.30 Lines ‖ to same line are ‖ to each other

1.33 Lines joining equal and ‖ lines are themselves equal and ‖

1.34 ‖gm: equal opp ∡s and equal sides, and self-bisected by diagonal

Equal Areas

1.35 ‖gms on same base between same ‖s are equal

1.36 ‖gms on equal bases between same ‖s are equal

1.37 △s on same base between same ‖s are equal

1.38 △s on equal bases between same ‖s are equal

1.39 Equal △s same side of same base are between same ‖s

1.40 Equal △s on equal bases on same side of same line are between same ‖s

1.41 if ‖gm and △ on same base between same ‖s, ‖gm double

1.43 Complements about diagonal of ‖gm are equal

Book II

Definitions

1. Every **rectangle** is contained by two sides enclosing a ⊾
2. In ‖gm, a ‖gm about its diagonal plus the two complements is a **gnomon**.
3. When a line is divided into parts, each part is a **segment**. If within original line, **internal**. Else, **external**.

Constructions

2.11 Divide AB in 2 parts @H: AB•HB=AH^2

Algebra

2.1 AB•CD = AB•(segments of CD)

2.2 AB•(segments AB) = AB^2

2.3 AB cut @ C, AB•BC = BC^2 + AC•CB

2.4 AB cut @ C, AB^2 = AC^2 + CB^2 + 2 AC•CB

2.4.C1 ‖gms on diagonal of square are squares

2.4.C2 Squares on 2AB = 4(AB^2)

2.5 AB, bisect C, D on CB, AD•DB + CD^2 = CB^2 = AC^2

2.6 AB, bisect C, produce BD, AD•DB + CB^2 = CD^2

2.7 AB cut C, AB^2 + CB^2 = 2(AB•CB) + AC^2

2.8 AB cut C, (AB+CB)^2 = 4(AB•CB) + AC^2

2.9 AB bisect C, D on CB, AD^2 + DB^2 = 2(AC^2 + CD^2)

2.10 AB bisect C, produce BD, AD^2 + DB^2 = 2(AC^2 + CD^2)

2.12 △ABC, ∡C obtuse, BC produced meets AD⊥BD, AB^2 = AC^2 + BC^2 + 2(BC•CD)

2.13 △ABC, ∡B acute, AD⊥BC, AC^2 = AB^2 + BC^2 - 2(BC•BD)

2.13.n1 ABC, median AD, AB^2 + AC^2 = 2((1/2BC)^2 + AD^2)

Book III

Definitions

1. 2 ○ **equal** if diameters or radii equal.
2. A line touches a ○ if it meets a ○ and if produced does not cut it. This is a **tangent** with its point of contact.
3. 2 ○ **touch** if they meet but do not cut. If ○ A in ○ B, A touches **internally**, else **externally**.
4. A line cutting a ○ at 2 points is a **secant**.
5. A **chord** is a line connecting two points of a ○.
6. **Chords' distances** are measured by their ⊥s to the center.
7. A **segment** of a ○ is a chord and what it cuts off. The chord is the segment's base.
8. The **∡ of a segmen**t is the ∡ from any point of the ○ whose arms extend to a segments endpoints and insists or stands upon the part of the ○ between the arms.
9. Any part of a ○'s boundary is an **arc**.
10. A **sector** is a figure bounded by two radii and the interceptted arc. The ∡ of the radii is the **sector's ∡**.
11. 2 ○ with the same center are **concentric**.

Constructions

3.1 Given ○, find center

3.17 From point, on or outside ○, draw tangent.

3.25 Given arc, create its ○.

3.30 Bisect a given arc.

3.33 Given line, ∡, create ○ segment containing ∡ equal given ∡.

3.34 Given ○, ∡, cut segment containing ∡.

Circles

3.2 Lines joining 2 points on ○ lie within circle

3.3 If line through ○ center bisects chord, it cuts at ⊾, and vice versa

3.4 2 chords, not both through center cannot bisect each other

3.5 If 2 ○ cut each other, not same center

3.6 If 2 ○ touch internally, not same center

3.7 ○ABCDG, center E, diam AD, F on ED, of Fx, 1)FA greatest, 2)FD least, 3) nearer FA > more remote, 4) G on circle, only one line equal FG possible

3.8 ○ACB, diam BA produced to D outside, C at π, 1) lines Dx on concave arc AC, DC<Dx<DA 2) lines on convex arc C'B, DC'>Dx>DB 3) For Dn to ○, only one equal Dm possible

3.9 ○(D,DA): if more than two equal lines from E in ○ to ○, E=D

3.10 ○ cannot ○ cut at more than 2 points

3.11 If 2 ○ touch internally, line through centers includes point of contact.

3.12 If 2 ○ touch externally, line through centers includes point of contact.

3.13 Internally or externally 2 ○ touch at exactly one point.

3.14 Equal chords are equidistant from center and conversely.

3.15 Diameter is greatest chord. Chords nearer center are greater than those more remote.

3.16 Line ⊥ to end of diameter lies outside ○.

3.16.C1 Line ⊥ to end of diameter touches ○.

3.16.C2 Tangent touches ○ at exactly one point

3.16.C3 For any point on ○ there exists exactly one tangent.

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Books III - V

Circles (contd)
3.18 Radius to tangent is ⊥ to tangent
3.19 Line ⊥ to point of tangency includes center.
3.20 ∡ from center is 2∡ from circle on same arc.
3.21 ∡s in same arc on same chord are equal
3.22 4-gon in ○, opp ∡s = 2⊾ and conversely.
3.23 Same side of same chord, similar arcs coincide.
3.24 Converse of 3.23
3.26 In equal ○s, arcs on equal ∡s, from center or circle, are equal.
3.27 Converse of 3.26
3.28 In equal ○s, arcs on same side of equal chords are equal
3.29 Converse of 3.28
3.31 ∡ on semicircle is ⊾; on greater arc, obtuse; on lesser, acute
3.31.C1 If one ∡ of a △ equals the other two ∡s, it is a ⊾.
3.32 Given chord from tangent's point of contact, ∡s chord to tangent equal ∡s of alternate segments
3.35 In ○, if two chords intersect, rectangle of one chord's segments equals the rectangle of the others'.
3.36 If from point outside ○, one line is drawn to touch ○ and one to cut it, the square of the first equals the rectangle of the second and its outside segment.
3.36.C1 Given secants from point outside ○, the rectangles of their whole and outside segments are equal.
3.37 If from point outside ○, one line is drawn to meet ○ and one to cut it and the square of the first equals the rectangle of the second and its outside segment, the first is tangent.

Book IV
Definitions
1. One rectilineal figure is **inscribed** in another if all its ∡s touch the other's sides.
2. The outer figure is then said to be **circumscribed** about the inner.
3. A rectilineal figure is inscribed in a ○ if all its ∡s touch the ○.
4. A rectilineal figure is circumscribed about a ○ if all its sides are tangents.
5. A ○ is inscribed within a rectiineal figure if it touches all the figure's sides. [Note: ○ is **escribed** to a △ if it touches one side and the other two, produced.]
6. A ○ is **described** about a rectliineal figure if all the figure'∡s s are on the ○.
7. A line is **placed** in a ○ when it forms a chord.
8.A rectilineal figure w/ > 4 sides is a **polygon** (5:**penta-**, 6:**hexa-**, 7:**hepta-**, 8:**octa-**, 10:**deca-**, 12:**dodeca-**, 15:**quindeca-**)
9. A **regular polygon** has equal ∡s and sides.

Constructions
4.1 Given ○, line < diameter, draw chord equal to line.
4.2 Given ○, △, inscribe △ of equal ∡s to given △.
4.3 Given ○, △, circumscribe △ of equal ∡s to given △.
4.4.N1 Given △, create escribed ○.
4.4.N1.C1 Line from center to apex bisects base and apex's ∡.
4.5 Given △, describe a ○ about it.
4.5.C1 If △ acute, ○'s center in △; if right, center on hypotenuse; if obtuse, center outside opp obtuse ∡.
4.6 Given ○ inscribe square.
4.7 Given ○ describe square.
4.8 Given square, inscribe ○.
4.9 Given square, describe ○.
4.10 Create isosceles △ with vertex ∡ = 2 base ∡.
4.11 Given ○, inscribe regular 5-gon.
4.12 Given ○, describe regular 5-gon.
4.13 Given regular 5-gon, inscribe ○.
4.14 Given regular 5-gon, describe ○.
4.15 Given ○, inscribe regular 6-gon.
4.16 Given ○, inscribe regular 15-gon.

Book V
Definitions
1. A lesser magnitude is an **aliquot part**, **measure**, or **submultiple** of a greater if the greater contains the lesser an exact number of times.
2.The greater is then a **multiple** of the lesser.
3. **Ratio** is the relation of two magnitudes in terms of quantity. First term of A:B is **antecedent**, second is **consequent**.
4. Magnitudes may only have a ratio if they are of the same kind.
5. In the ratio A:B::C:D, for any m,n in **N**, n<m: nA<mB and nC<mD, n=m: nA=mB and nC=mD, n>m: nA>mB and nC>mD,
6. Magnitudes of the same ratio are **proportionals**. With 4 magnitudes as above, then A is to B as C is to D. A,D are the **extremes**, B,C the **means**.
7. If in proportionals nA>mB, C≤mD, A has a **greater ratio** to be than C to D and C has a **lesser ratio** to D than A to B.
8. **Proportion** (or **analogy**) is the similitude of ratios.
9. Proportions have at least 3 terms.
10. Such are in **common proportion** when A:B::B:C, B:C::C:D, C:D::D:E, and so on. Given 3 such magnitudes, A has a **duplicate ratio** to C, given 4, A has a **triplicate ratio** to D.
11. Given n magnitudes (m(i)), m(1) is in **compound proportion** to m(n) compounded of m(1):m(2), m(2):m(3), …, m(n-1):m(n).
12. Proportion's antecedents are **homologous** to each other and consequents are homologous to each other.
13. **Permuted** or **alternated**: A:C::B:D
14. **Inverted**: B:A::D:C
15. **Compounded**: A+B:B::C+D:D
16. **Divided**: A-B:B::C-D:D
17. **Converted**: A:A-B::C:C-D
18. **By equality** means, given set a of n magnitudes and sets b,c,... of n magnitudes, then a(1):a(n)::a(1):b(n)::a(1):c(n) ... Of this there are two kinds.
19. **Direct equality** means, given A,B,C,... and P,Q,R,... if A:B::P:Q and B:C::Q:R, then A:C::P:R.
20. **Disordered**, **perturbed equality** or **cross-order** means if A:B::Q:R and B:C::P:Q then A:C::P:R

Axioms (Simson)
1. Equimultiples of same or equal magnitudes are equal.
2. Magnitudes, of which same of equal equimultiples are equimultiples, are equal to each other.
3. A multiple of a greater magnitude is greater than the same multiple of a lesser.
4. That magnitude, of which a multiple is greater than the same multiple of another, is greater than that other.

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Books V - VI

Propositions
(magnitude A, number a)
5.1 If A=mE, B=mF, C=mG,
then A+B+C = m(E+F+G)
5.2 If A=mB, C=mD, E=nB, F=mD,
then A+E=(m+n)B, C+F=(m+n)D
5.3 If A=mB, C=mD,
then nA=nmB, nC=nmD
5.4 If A:B::C:D, m,n in **N**,
then mA:nB::mC:nD
5.5 If A=mB, C=mD,
then A-C = m(B-D)
5.6 If A=mC, B=mD, E=nC, F=nD,
then A-E=(m-n)C and B-F=(m-n)D
5.A If A:B::C:D,
the A>=<B as C>=<D
5.B If A:B::C:D, then B:A::D:C
5.C If A=mB, C=mD, then A:B::C:D
5.D Converse of 5.C
5.7 If A=B, then A:C::B:C and C:A::C:B
5.8 If A>B, then A:C>B:C and C:B<C:A
5.9 If A:C::B:C then A=B and conversely
5.10 If A:C>B:B then A>B and
if C:B>C:A then B<A
5.11 If A:B::C:D and C:D::E:F
then A:B::E:F
5.12 If A:B::C:D::E:F
then A:B::A+C+E:B+D+F
5.13 If A:B::C:D and C:D>E:F
then A:B>E:F
5.14 If A:B::C:D then
A>=<C as B>=<D
5.15 A:B::mA:mB
5:16 If A:B::C:D, then A:C::B:D
5.17 If A:B::C:D, thenA-B:B::C-D:D
5.18 If A:B::C:D, thenA+B:B::C+D:D
5.19 If A:B::C:D, then A-C:B-D::A:B
5.E If A:B::C:D, then A:A-B:C:C-D
5.20 Any ABC, DEF, if A:B::D:E and
B:C::E:F then A>=<C as D>=<F
5.20 Any ABC, DEF, if A:B::E:F and
B:C::D:E then A>=<C as D>=<F

5.22 Given sets A,B of n magnitudes such that A(i):A(i+1)::B(i):B(i+1)
then A(1):A(n)::B(1):B(n)
5.23 Given sets A,B of n magnitudes such that A(i):A(i+1)::B(i+1):B(i+2) and A(i+1):A(i+2)::B(i):B(i+1),
then A(1):A(n)::B(1):B(n)
5.F By 5.22,23, ratios compounded of equal ratios are equal.
5.24 If A:B::C:D and E:B::F:D.
Then A+E:B::C+F:D
5.24 If A:B::C:D and A greatest magnitude, then A+D>B+C

Book VI
Definitions
1. Two rectilineal figures are **equiangular** if their angles, taken in the same order, are equal.
2. **Similar** figures are equiangular and their sides, taken in the same order, are proportional. Corresponding sides are **homologous** (precendents/antecedents in ratios)
3. **Reciprocal** figures (always △) share two angles, the enclosing sides of which are proportional.
4. AB cut @ C is in **extreme and mean ratio** when AC<CB, AB:AC::AC:CB
5. The **altitude** (altd.) of a figure is a line from its vertex (highest point) to the base.

Theorems
6.1 △ and ∥gms of same altd are to one another as their bases.
6.2 Line ∥ to side of △ will proportionately cut other sides (produced if necessary) and conversely.
6.3 Bisector of △ apex cuts base into segments proportional to sides.
6.A Bisector of △ ext∡, base produced and produced proportional to sides.
6.4 Two △ equiangular, enclosing sides of ∡ angle on one △ proportional to enclosing sides of other △.
6.4.C1 Equiangular △s are similar
6.5 If the sides about the ∡s of two △s taken in order are proportional, the △s are equi∡.
6.6 If two △s share one ∡ with proportional enclosing sides, the △s are equi∡.
6.7 If two △s share an ∡, with proportional enclosing sides on 2d ∡, the 3d ∡s are either equal or supplementary.
6.8 Given ⊾△, and ⊥ from ⊾ to base, the given △ and two created are all similar to each other.
6.8 C1 a. ⊥ is mean proportional of base segments. b. Each side of original △ is mean proportional of base and adj. segment.
6.14 ∥gms of equal area sharing ∡ have proportional sides about equal ∡s. And conversely.
6.15 △s of equal area sharing ∡ have proportional sides about equal ∡s. And conversely.
6.16 If AB:CD::EF:GH then AB•GH = CD•EF and conversely.
6.17 If AB:CD::CD:EF then AB•EF=CD^2 and conversely
6.19 Similar △s are in duplicate ratio of their homologous sides.
6.20 Similar n-gons can be divided into equal number of similar △s of same ratio to each other as n-gons to each other and n-gons are in duplicate ratio of their homologous sides.
6.20.C1 Similar rectilineal figures are in duplicate ratio of their homologous sides.

6.20.C2 Given three lines: A:B::B:C, n-gon on A:similar n-gon on B::A:C (duplicate ratio)
6.20.C3 Given line A:B::B:C, A:C::A^2:B^2
6.20.C4 Similar rectilineal figures are to each other as the squares on homologous sides.
6.21 N-gons similar to the same n-gon are similar to each other.
6.22 Given lines AB:CD::EF:GH, any similar n-gons on AB, CD are proportional to any other similar n-gons on EF, GH
6.23 Equi∡ ∥gms are proportional to the compound ratio of their sides
6.24 ∥gms on diagonal of ∥gm are similar to each other and to the whole
6.26 If two similar ∥gms have a common ∡ and same orientation, thay are on the same diagonal.
[6.27-29 ellided]
6.31 Given ⊾△, any n-gon on hypotenuse equals sum of similar n-gons on sides.
6.32 If two △s have two proportional sides and are joined such that homologous sides are ∥, remaining sides are on one line
6.33 In equal ○s, ∡s, on center or on ○, have the same ratio as the arcs subtended. Same for sectors.
6.B For any △ with apex ∡ bisected, rectangle of sides equals bisector^2 plus rectangle of bisector's segments of base
6.C For any inscribed △ with line from apex ⊥ base, rectangle of sides equals rectangle of ⊥ and diameter of ○.
6.D For any 4-gon inscribed in ○, rectangle of diagonals equals sum of rectangles of opp sides.

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Books VI, XI (1-21), XI (1-2)

Constructions
6.9 Given AB, cut off given submultiple.
6.10 Given divided AB, divide CD similarly.
6.11 Given 2 lines, find 3d proportional.
6.12 Given 3 lines, find 4th proportional.
6.13 Given 2 lines, find mean proportional.
6.18 Given n-gon and line, construct similar n-gon on line with same orientation
6.25 Given two n-gons, describe a third similar to the first and equal to the second.
6.30 Divide given line into extreme and mean ratio.

Book XI
Definitions
1. A **solid** has length, breadth, and thickness.
2. A solid is bounded by a **surface**.
3. A line is **perpendicular or a normal to a plane** if it is at ⊾ to every line in the plane meeting it.
4. **Planes are perpendicular** when lines ⊥ to their intersection lie in the other plane.
5. **Angle of line to plane** is the acute ∡ between that line and a line from its point of intersection with the plane to a normal from line to plane.
6. **Angle of planes** is the acute ∡ of two lines, one in each plane, from a point on the intersection of the planes.
7. Two planes have the same angle as two other planes when their angles of planes are equal.
8. **Parallel planes** do not meet if produced. A line is ∥ to a plane if they do not meet when produced.
9. A **solid angle** is the ∡ of three or more planes meeting at a point. If three, angle is **trihedral**. If more, **polyhedral**.
10. The angle of two lines which do not meet is the angle of their parallels which do meet.
11. **Similar solid figures** are equiangular and contained by equal numbers of planes.
12. A **polyhedron** is a solid figure bounded by planes. It is **regular** when bounded by equal regular n-gons.
13. A **pyramid** has any n-gon for a base and triangles for sides which have edges of the n-gon for a base and whose apexes meet at a point.
14. A **prism** has two opposite, equal, parallel n-gon surfaces. The remaining surfaces are parallelograms.
15. A **sphere** is the revolution of a semicircle about a fixed diameter.
16. The **axis of a sphere** is its fixed diameter of revolution.
17, The **center of a sphere** is that of its semicircle. Its **diameter** is any line through its center, terminated on its surface.
18.A **right circular cone** is a right triangle rotated about its side. If that side is equal to the other, the cone is **right-angled**, if less, **obtuse-angled**, if more, **acute-angled**.
19. **Axis of a cone** is its line of revolution.
20. **Base of a cone** is described by its other side.
21. A **right circular cylinder** is a rectangle in revolution.
22. Its **axis** is the side of revolution.
23. Its **bases** are the circles described by opposite sides.
24. **Similar cones and cylinders** have proportional axes and base diameters.
25. A **cube** is contained by 6 equal squares.
26. A **tetrahedron** is contained by 4 triangles, which is equal and equilateral make it **regular.**
27. A regular **octahedron** is contained by 8 equal, equilateral triangles.
28. A regular **dodecahedron** is contained by 12 equal, equilateral, equiangular pentagons.
29. A regular **icosahedron** is contained by 20 equal, equilateral triangles.
30. A **parallelipiped** is contained by 6 4-gons and each pair of opposite sides are parallel.
31. The **projection** of a line on a plane is the sum of its perpendiculars' intersections on the plane.

Propositions
1. If one part of a line is in a plane, another part cannot be out of it.
2. Two intersecting lines or three lines which meet lie in one plane.
3. The intersection of two planes is a line.
4. Let a line be at ⊾ to the point of intersection of two other lines, then it is ⊥ to their plane.
5. If 3 lines meet at a point and a fourth is ⊥ to all three, the 3 lie in one plane.
6. If 2 lines are ⊥ to the same plane they are ∥.
7. If two lines are ∥, any line joining them lies in their plane.
8. If two lines are ∥ and the first is ⊥ to a plane, so is the second.
9. Two lines, each ∥ to a line in another plane, are ∥ to each other.
10. If two lines intersecting in one plane are ∥ to two lines intersecting in another, both pairs contain equal angles.
13. From point on plane there can be only one ⊥ on same side and only one ⊥ from point not on plane
14. Planes ⊥ to same line are ∥ to each other.
15. If two intersecting lines are ∥ to two intersecting lines in another plane, the two planes are ∥.
16. If two ∥ planes are cut by a third, the two intersections are ∥
17. Two lines cut by ∥ planes are cut in the same ratio.
18. If a line ⊥ to plane, every plane through that line is ⊥ to that plane.
19. If two intersectiong planes are ⊥ to a third, their intersection is ⊥ to the third.
20. If a solid ∡ is containged by 3 plane ∡s, any 2 > 3rd.
21. Every solid ∡ is contained by plane ∡s together less than 4 ⊾.

Constructions
11. Given plane and point not on plane, create ⊥ from point to plane
12. From point on plane, create line ⊥ to plane.

Book XII
Propositions
Lemma (X.1) Given 2 magnitudes, by repeatedly removing half or more of the greater, it shall be smaller than the lesser.
1. Similar inscribed n-gons are in the proportions of the squares **on** the diameters.
2. ○s are to one another as the squares **on** their diameters

Euclid – Todhunter – Synopsis

Immediate Results Following Euclid

Lines
1. Shortest line from point to other line is ⊥.
2. Given ∡BAC, its bisector AD, ⊥s from AD to AC, AB equal.
3. Lines ⊥ to same line are ∥.
4. From any point equidistant from 2 ∥ lines, any 2 lines cutting the ∥ lines will intercept equal portions of them.
5. If 2 lines cut by 3 ∥ lines, intercepts on 2 lines proportional.

Triangles
1. Any 2 △ with two equal ∡s, 3d ∡s equal..
2. Difference of any 2 sides is less than 3d side.
3. Given △ and any point, sum of distances from ∡s to point > ½ sum of sides
4. Any 2 sides greater than twice median from their enclosed ∡.
5. Sum of 1 ∡ = other 2, ⊾△, < other 2, acute △, > other two, obtuse △.
6. Line ∥ 1st side, through midpoint of 2d, bisects 3d.
7. Any△ bisected by its medians.
8. Line joining midpoints of sides = ½ base and is ∥ to base and cuts off ¼ △.
9. If 2 sides given, area maximized if enclosed ∡ is ⊾.
10. 4(sum squares on medians) = 3(sum squares on sides)
11. ∡s of equi∡ △ = 2/3 ⊾.
12. equi∡△, square on median is 3 times square on ½ base.
13. ⊾△ median from ⊾ = ½ hypotenuse.
14. ⊾△ABC, AD⊥BC, AD^2 = BD•DC and AC^2 = BC•CD
15. If inscribed and described ○s concentric, △ equi∡
16. If 2 △s equi∡, sides proportional and conversely.
17. Line ∥ base cuts off similar △.
18. Any △, join apex to base, inscribe resulting △s, diameters proportional to △s sides.
19. 2 equal △s, opp same base, line joining vertices bisected by base (produced).
20. Median bisects all lines through sides ∥ to base.
21. ⊥s from mdpts of sides meet @ point.
22. Medians meet @ point (**centroid**)
23. Bisectors of △s meet @ point.
24. Lines ⊥ to ∡s' vertices meet @ point (**orthocenter**)
25. If two medians are equal, their ∡s are equal.
26. Difference of squares on sides = 2(base x projection of apex's median on base)

Isosceles △
1. If median from vertex ⊥ base, △ isosceles and conversely.
2. ⊥s from sides into base ∡s equal.
3. ⊥ from vertex to base bisects base and vertex ∡.
4. △ABC, any D on base BC, BD•DC = $AC^2 – AB^2$
5. If base ∡ = 2 apex ∡,apex ∡ = 1/5 2⊾.
6. Greatest area of all △s of equal perimeter.

Parallelograms
3. Diagonals of ∥gm bisect each other and conversely.
4. In ∥gm, if diagonals bisect opp ∡s, **rhombus**.
5. In ∥gm, lines bisecting adj ∡s, intersect at ⊾.
6. In ∥gm, if diagonals equal, then ∡s equal and **rectangle**.
7. In ∥gm, line through intersection of diagonals and ∥ to side, bisects ∥gm
8. In ∥gm, diagonals create 4 △s of equal area.
9. In ∥gm, sum squares on diagonals = sum squares on sides.

N-gons
1. Sum of int ∡s of n-gon = (2n-4) ⊾. Sum of ∡s of 4-gon = 4 ⊾.
2. Each ∡ of an equi∡ n-gon = (2n-4)/n ⊾.
3. Regular 5-gon, ∡ trisected by diagonals to opp ∡.
4. Regular 5-gon, diagonals describe regular 5-gon.
5. Regular n-gon, ∡s bisectors meet @ point.
6. Area of regular 6-gon is twice area equi∡ △ inscribed in same ○.
7. Equilateral figure inscribe in circle is equi∡.
8. Regular n-gon, center of inscribed, described circles is intersection of bisectors of 2 adj ∡s.
9. Regular inscribed n-gon, tangents at corners form reqular n-gon.

Quadrilaterals (4-gon)
1. Sum of ∡s = 4 ⊾.
2, If opp ∡s equal, each to each, **∥gm**.
3. If opp sides equal, each to each, **∥gm**.
4. Lines joining midpoints of adj sides creates ∥gm
5. Sum squares on sides = sum squares on diagonals + 4(square on line joining midpoints of diagonals)
6. If diagonals bisect e.o. @ ⊾, **rhombus** and conversely.
7. Opp. ∡ of **rhombus** are equal and bisected by diagonals.
8. In **rhombus**, diagonals at ⊾.
9. Of all rectangles of same perimeter, **square** has greatest area.
10. If diagonals equal and bisect at ⊾, **square**.
11. Square on diagonal of square is twice **square**.
12. 2. If 4-gon circumscribes ○, sum of opp sides equal and conversely.
13. Diagonals of a trapezium cut e.o. in the ratio of the ∥ sides.
14. Trapezium area = alt•(sum of ∥ sides)

Euclid – Todhunter – Synopsis

Immediate Results Following Euclid

Circles	Planes	Solids	
1. 2 ○s meeting at 2 points, line between centers bisects line between points at ⊾. 2. ∥ chords are bisected by the diameter passing through them at ⊾. 3. Midpoints of all equal chords lie on a concentric ○. 4. Three non-linear points determine a ○. 5. If distance between centers of 2 ○s equal sum of radii, ○s touch externally, if equal to difference of radii, internally. 6. If ○ is tangent to two lines, its center lies on their bisector. 7. Tangents on chord meet on radius produced. Let tangents meet @ T, chord BC, center A. then $CN \cdot CT = CA^2$ 8. Tangents ∥, then tangencies on diameter. 9.Let AB,CD meet at O, If AO•OB = CO•OD, ABCD on circle. 10. If 2 ○s intersect, tangents from common chord produced are equal and common chord bisects common tangent. 11. Incribed square is double square on radius. 12. Described square is double inscribed square. 13. If 2 ○s touch each other and line, let A=diam 1, B=diam 2, C=seegment between tangencies, A:C::C:B, C mean proportional. 14. 2 chords intersect inside, ∡ is ½ sum of intercepted arcs. 15. 2 chords intersect outside, ∡ is ½ difference of intercepted arcs.	1. ∡ between 2 planes is ∡ between their ⊥s. 2. Lines between point and plane, ⊥ is shortest and of other lines from that point ones closer to foot of ⊥ are shorter than those remote. 3. Line ∥ to another line is ∥ to all planes passing through that line. 4. If ⊥ on 2 points of plane be equal, line on extremeties ∥ to plane. 5. Equal lines from point to plane form equal ∡s to plane. 6. 2 planes not ∥, cur by 2 ∥ planes, lines of intersection contain equal ∡s.	1. Tetrahedron, sum of squares on opp edges, less than sum of squares on other 4 edges. 2. Tetrahedron, sum of squares 6 edges = 4(sum squares lines joining mdpts opp edges) 3. Tetrahedron, mdpts 2 pairs of opp edges lie on same plane and form ∥gm. 4. N-gons formed by cutting prism with ∥ planes are equal	